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# Coupled Collective and Rabi Oscillations Triggered by Electron Transport through a Photon Cavity

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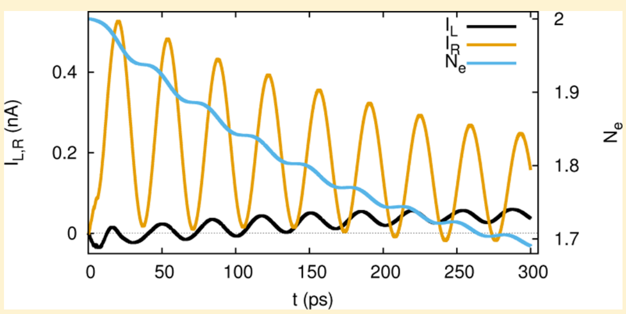


**ABSTRACT:** We show how the switching on of electron transport through a system of two parallel quantum dots embedded in a short quantum wire in a photon cavity can trigger coupled Rabi and collective electron−photon oscillations. We select the initial state of the system to be an eigenstate of the closed system containing two Coulomb-interacting electrons with possibly few photons of a single cavity mode. The many-level quantum dots are described by a continuous potential. The Coulomb interaction and the para- and diamagnetic electron−photon interactions are treated by exact diagonalization in a truncated Fock space. To identify the collective modes, the results are compared for an open and a closed system with respect to the coupling to external electron reservoirs, or leads. We demonstrate that the vacuum Rabi oscillations can be seen in transport quantities as the current in and out of the system.

**KEYWORDS:** *Rabi oscillations, electron transport, collective oscillations, transient, double quantum dots*

Fine-tuning of the electron−photon interaction has opened up new possibilities in semiconductor physics. The transport of electrons through quantum dots assisted by up to four photons in the terahertz frequency range has been observed,[1] and double quantum dots have been used to detect single photons from shot noise in electron transport through a quantum point contact.[2] The properties and control of atomic or electronic systems in photonic cavities is a common theme in the research effort of many teams working on various aspects of quantum cavity electrodynamics and related fields.[3−10] The nonlocal single-photon transport properties of two sets of double quantum dots within a photon cavity have recently been modeled,[11] and a pump−probe scheme for electron−photon dynamics in a hybrid conductor−cavity system with one electron reservoir was also investigated.[12] Many tasks in quantum information processing might be served by mixed photon−electron circuits. To model such systems, we need to combine methods and tools that have traditionally been used and developed in the fields of time-dependent electron transport and quantum optics. In this publication, we show how time-dependent electron transport through a nanoscale system embedded in a photon cavity could be used to detect vacuum Rabi oscillations in it. To do so, we use a generalized master equation (GME) formalism for time-dependent electron transport that was initially developed for quantum optics systems.[13,14]

## THE CLOSED SYSTEM IN EQUILIBRIUM

We consider a 2D electron system lying in the $xy$ plane (GaAs parameters, $\kappa = 12.4$ and $m^* = 0.067m_e$) that is subject to a homogeneous external weak magnetic field in the $z$ direction ($B = 0.1$ T). The system represents a short quantum wire with parabolic confinement in the $y$ direction, with energy $\hbar\Omega_0 = 2.0$ meV but hard walls in the $x$ direction. Two shallow parallel quantum dots are embedded in the wire as is illustrated in Figure 1. The external magnetic field and the parabolic confinement define the natural length scale $a_w = (\hbar/(m^*\Omega_w))^{1/2}$, with $\Omega_w = (\Omega_0^2 + \omega_c^2)^{1/2}$, where $\omega_c = (eB/(m^*c))$. The Coulomb interaction of the electrons in the system is considered using configuration interaction in a truncated Fock space. The 2D electron system is placed in a photon cavity with one mode of energy $E_{EM}$ and linear polarization in the $x$ or $y$ direction. For the electron−photon interaction, we retain both the para- and the diamagnetic terms without the rotating wave approximation, but consider the wavelength much larger than the size of the electron system.[15,16] The two parts of the electron−photon interaction are used because we consider the system both to be on- and off-resonance.[17] We begin by using the electron−photon coupling strength $g_{EM} = 0.05$ meV. Not all types of cavities may admit an external perpendicular magnetic field. We keep it in the model

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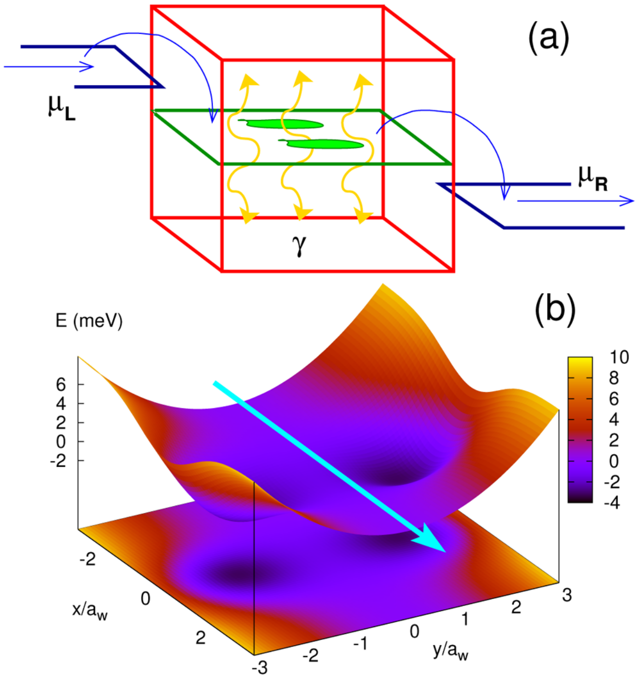


**Figure 1.** (a) Schema of the leads−wire−cavity system. The chemical potentials are indicated for the leads as well as the photon mode for the central system (wavy vertical arrows). (b) Potential landscape defining the two parallel quantum dots in a short parabolically confined quantum wire. The arrow (cyan) indicates the general direction ($x$ direction) of electron transport after the system has been opened up. Effective magnetic length $a_w$ = 23.8 nm, $\hbar\Omega_0$ = 2.0 meV, and $B$ = 0.1 T.

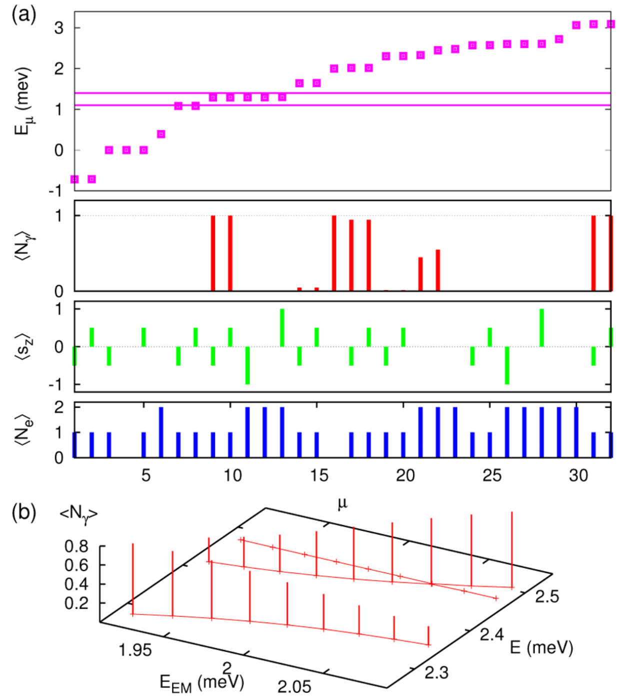


**Figure 2.** (a) Lowest part of the energy spectrum $E_\mu$ (squares, units of meV) for the closed system vs state number $\mu$. The photon content $\langle N_\gamma \rangle$ of the states is indicated with vertical red bars, the spin $\langle s_z \rangle$ (units of $\hbar$) of each state are indicated with green bars, and the electron content $\langle N_e \rangle$ is indicated with blue bars. The photons are $y$-polarized with energy $E_{EM}$ = 2.0 meV. The two horizontal lines (pink) indicate the chemical potentials of the biased leads that are coupled to the system to open it up to electrons, as discussed further in the text. (b) The Rabi vacuum splitting of the two-electron states $|\breve{21})$ and $|\breve{22})$. The photon content is indicated with red bars. The two-electron state $|\breve{23})$ with vanishing photon content enters the Rabi-splitting regime and participates in the transport. $g_{EM}$ = 0.05 meV.

in order to take proper care of the spin degree of freedom in the numerical calculations and to track possible effects of the coupling of the electron motion along or perpendicular to the short quantum wire.

The energy spectrum of the closed system is displayed in Figure 2a together with information about the electron, photon, and spin content of the lowest eigenstates for a photon field with $y$ polarization and energy $E_{EM}$ chosen close to the confinement frequency $\hbar\Omega_0$. We obtain a vacuum Rabi splitting for the two-electron state containing one photon, resulting in the Rabi pair ($|\breve{21})$, $|\breve{22})$) seen in Figure 2b. We denote by $|\breve{\mu})$ the composite many-body electron−photon eigenstates.[15,16]

## THE CLOSED SYSTEM OUT OF EQUILIBRIUM

We now consider a short classical electromagnetic pulse perturbing the closed system. The time-evolution of the system is calculated by direct integration of the Liouville−von Neumann equation for the density matrix.[18,19] We start the time-evolution for the system in two different states, with the cavity photons having either $x$ or $y$ polarization, and with the excitation pulse with the same polarization as the photons. In the $x$ polarization case, we use the two-electron ground state $|\breve{6})$. For the $y$ polarization, we select the Rabi-split state with the higher photon content ($\gtrsim$ 0.5). After the excitation pulse has vanished, the occupation is constant and is seen in Figure 3a,b for the two cases. The pulse is shown in the inset of Figure 3c (red curve).

The former excitation ($x$ polarization) gives a gapped spectrum for which most transitions can be related to known dipole-active many-body states.[20] The latter excitation ($y$ polarization) is very close to a resonance in the system and results in the activation of many transitions visible in Figure 3b. More important is the fact that this type of low-frequency excitation pulse not only causes the occupation of the other Rabi-vacuum-split state, i.e., $|\breve{21})$ together with $|\breve{23})$, but also a strong entanglement between the Rabi-vacuum components (signaled by large off-diagonal elements in the density matrix). The system is far from an eigenstate, and as displayed in Figure 4, it shows very strong pure Rabi oscillations in the mean photon number (Figure 4b) that are even present in the Fourier component of the expectation value of the center-of-mass $y$ coordinate (Figure 4a). If the photon energy is not in resonance with the confinement energy, then the excitation spectrum of the mean values of the center-of-mass coordinates is generally simpler for a not-too-strong excitation. This case was already accounted for in Figure 3a, where an $x$-polarized photon field is not in resonance with the electrons, higher states above the ground state are only slightly occupied, and no low energy modes are excited.

## THE OPEN SYSTEM

We have seen how an external electrical pulse can be used to excite the system out of a many-body eigenstate with a constant photon number into entangled states with an oscillating photon number. If we increase the frequency of the excitation pulse, then the two Rabi-split states get less entangled, and smaller

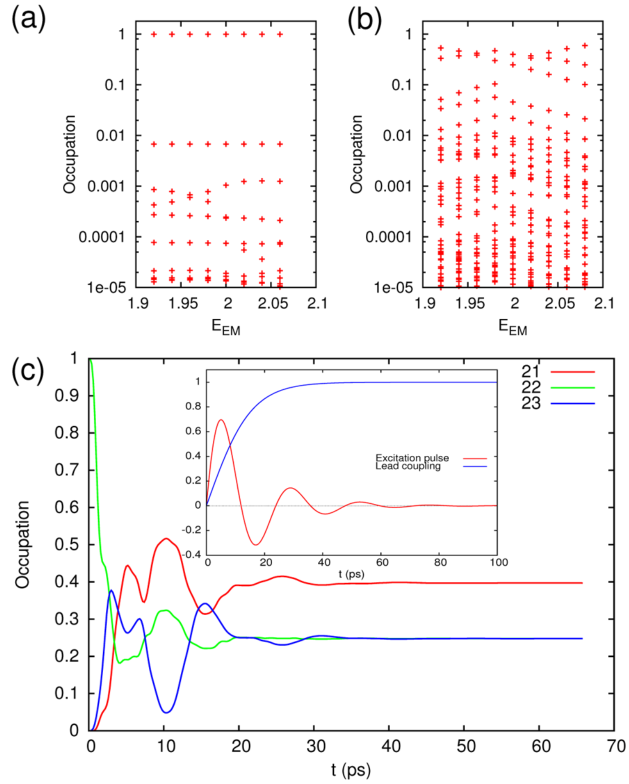


**Figure 3.** For the closed system, (a) the occupation of states $|\breve{\mu})$ for $x$ polarization and excitation in the $x$ direction. (b) Occupation for $y$ polarization and excitation in the $y$ direction. (c) Transient occupation for $y$ polarization and excitation in the $y$ direction for $E_{EM}$ = 2.0 meV. The inset shows the temporal part of the excitation pulse compared to the switching function for the lead coupling. The initial state is the lowest-energy two-electron Rabi-split state with photon content $\gtrsim$0.5 for the $y$ polarization but the two-electron ground state for the $x$ polarization.

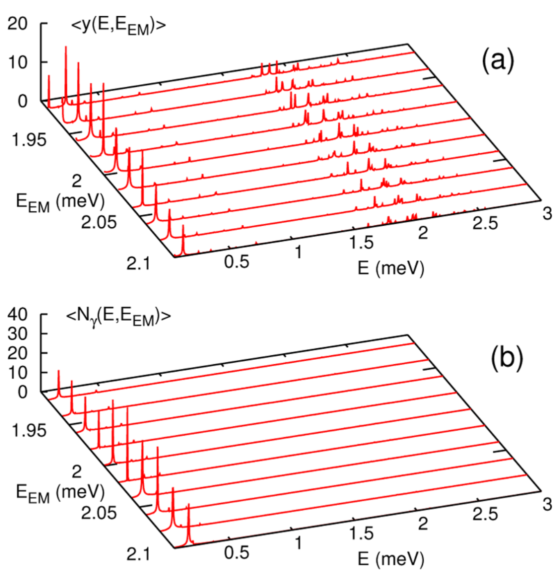


**Figure 4.** Fourier spectra in case of the closed system for (a) the center-of-mass coordinate $\langle y\rangle$ and (b) the mean photon number $\langle N_\gamma\rangle$ for the initial lowest-energy Rabi-split two-electron state with photon content $\gtrsim$0.5. $g_{EM}$ = 0.05 meV.

Rabi amplitude is observed. The question is thus what happens if instead of applying an electrical pulse, we gently open up the system for transport of electrons through it, and keep the coupling to the leads constant after the initial switch-on? The transport of electrons through a system in a photon cavity has been reported by Delbecq et al.[21] Can we then expect to see Rabi oscillations? To accomplish this, we describe the coupling of the system to two external parabolic semi-infinite leads with a non-Markovian GME, selecting a time-dependent coupling function shown in the inset of Figure 3c. The coupling function has a time scale similar to that of the external electrical pulse. The GME formalism with our spatially dependent coupling of states in the leads and the system has been described elsewhere.[15,16] (Here, the lead-system coupling strength is 0.5 meV and the lead temperature $T$ = 0.5 K.) The GME describes the time-evolution of the reduced-density operator of the central system under the influence of the external leads. It is derived by projecting the Liouville−von Neumann equation for the time evolution of the full-density operator for the system and the leads on the central system by tracing out variables of the leads. In addition to the unitary evolution of the reduced-density operator caused by the Hamiltonian of the central system including the coupling to the photons, the GME contains a complicated dissipation term with memory effects describing the tunneling of electrons between the central system and the leads. In our calculations, the GME leads to a coupled set of tens of thousands of integro-differential equations with time convolution caused by the non-Markovian memory effects.

The chemical potentials of the left (L) and right (R) leads, $\mu_L$ = 1.4 meV and $\mu_R$ = 1.1 meV, respectively, are chosen to include three two-electron and two one-electron states in the bias window, as indicated in Figure 2a. Because of the geometry of the system, the two-electron states have low coupling to the leads, resulting from their charge densities being low in the contact area of the central system. In the case of a $y$-polarized photon field approximately in resonance with the $y$ confinement potential, we observe small oscillations in the mean photon number seen in Figure 5a. The oscillations are small because the GME formalism as applied here is only valid for weak contacts to the leads. The frequency of the oscillations coincides with the Rabi frequency observed in the closed system and the Jaynes−Cummings model[22] when the states $|\breve{6})$ and $|\breve{22})$ (Figure 2a) are taken as the atomic states with our electron−photon coupling strength $g_{EM}$ = 0.05 meV and photon energy $E_{EM}$ = 2.0 meV.

We are here describing different ways to excite an electron−photon system confined by a continuous potential. To describe correctly the strong electron−photon interaction, we need a large basis in the Fock space built as a tensor product of Coulomb-interacting electron states and the eigenstates of the photon operator. As a byproduct, we can see collective oscillations emerging in the system opened up for transport, even in the weak coupling limit. In Figure 5b, we see the mean orbit of the center-of-mass of the two electrons for the two linear polarizations of the photon field. In both cases, the center-of-mass is shifted from the center of the system ($x$ = $y$ = 0) to the left as one of the electrons starts to seep slowly from the system into the right lead, performing revolutions that are synchronized with the oscillations of the photon number. We see effects of the weak magnetic field and the dissipation of energy to the leads. The occupation of the initial two-electron state is getting less probable, whereas lower energy one-electron states are gaining occupation probability. Figure 5b


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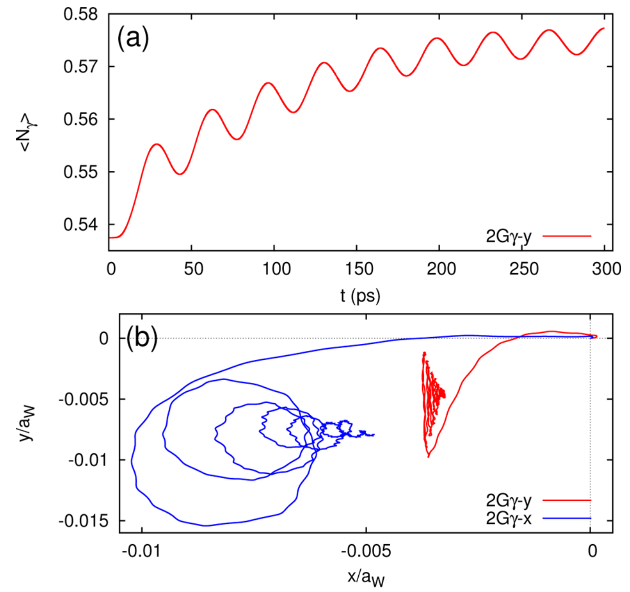


**Figure 5.** For the open system, (a) the mean photon number $\langle N_\gamma \rangle$ for $y$ polarization, and (b) the mean orbit of the center-of-mass of the initial lowest-energy Rabi-split two-electron state for $x$ polarization (blue) and $y$ polarization (red) with photon content $\gtrsim 0.5$ ($y$ polarization) and the lowest-energy two-electron one-photon state ($x$ polarization). $E_{EM}$ = 2.0 meV, $g_{EM}$ = 0.05 meV.

shows how the off-resonance system (blue curve) shows a simple spatially damped oscillation influenced by the magnetic field. In case of the Rabi resonance (red curve), the oscillation is almost entirely in the direction dictated by the electrical component of the photon.

The energy of the Rabi splitting as a function of the coupling constant $g_{EM}$ is compared for the open and the closed systems in Figure 6. The splittings for the open and the closed system

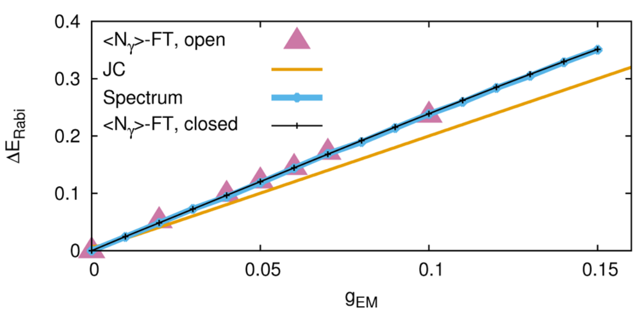


**Figure 6.** Energy of the Rabi splitting as found from the energy spectrum ("Spectrum"), the Fourier analysis of the oscillations in the mean photon number $\langle N_\gamma \rangle$ for the closed system ("$\langle N_\gamma \rangle$-FT, closed"), the open system ("$\langle N_\gamma \rangle$-FT, open"), and the two-level Jaynes–Cummings model ("JC"). $B$ = 0.1 T, $E_{EM}$ = 2.0 meV.

agree within the accuracy of the numerical calculations. They are a bit higher than the value known for the two-level Jaynes–Cummings model $\Delta E_{\rm Rabi}^{\rm JC} = ((\hbar\omega_r^2) + \delta^2)^{1/2}$, with the detuning $\delta$ = 7.44 $\mu$eV and the Rabi frequency $\hbar\omega_r = 2g_{EM}$ for the vacuum Rabi oscillations. This can be expected for a multilevel model.[23]

Because of the restriction of the GME formalism to weak contacts, the effects of the Rabi oscillations on the current in the leads is minor. However, if the system is initially excited by an external electrical pulse before it is opened up for transport, then the initial state for the transport would be a highly entangled state of the Rabi-split states, and the current in the leads would reflect that, as can be seen in Figure 7. The

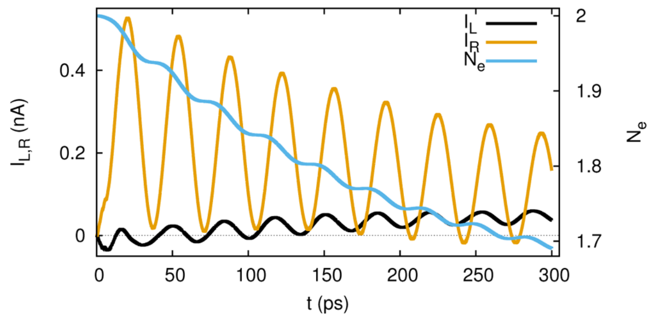


**Figure 7.** For the open system, the left and right currents (black and gold, respectively, left $y$ axis) and the mean number of electrons (blue, right $y$ axis) in case of full entanglement for the Rabi-split states $|\breve{2}1\rangle$ and $|\breve{2}2\rangle$ as initial states for a $y$-polarized photon field. $E_{EM}$ = 2.0 meV.

oscillations in the current caused by the vacuum Rabi oscillations decay with time as the occupation of the two-electron Rabi-split pair of states get less probable as charge enters and leaves the system and decoherence sets in.

## DISCUSSION AND SUMMARY

Effort has been put into guaranteeing the accuracy of the results presented here. The methods employed have been based on a grid-free numerical approach in an appropriate basis. We have used a so-called stepwise introduction of model complexities with the necessary truncation introduced elsewhere.[15] The needed basis size was in the range of 120–6000.

We show that even a weak contact of the central system to the external leads causes collective oscillations of the electrons and the photons in the system. Opposite to what happens in the closed system, the collective oscillations in the open system can change their character as the state of the central system evolves irreversibly in time. To describe the collective coupled oscillations of the strong interacting photons and electrons, it is necessary to resort to large bases of electron states and include both the para- and the diamagnetic interactions.

In the closed system, we observe strong vacuum Rabi oscillations in the photon content when the photon frequency is close to the parabolic lateral confinement frequency and its polarization is in the perpendicular direction ($y$ direction). This situation favors excitation by a low-frequency perpendicular electrical pulse because the vacuum Rabi splitting of the two-electron state is small. The excitation pulse then effectively puts the system into an entangled state of the two Rabi-split states.

The vacuum Rabi oscillation is also seen in the open system under the same initial conditions for the photon field, but its amplitude is small because the contacts to the leads are not very effective in forming an entangled state of the Rabi-split states. This can be enhanced by first exciting the system by an external electric pulse before it is opened up for electron transport. We have neglected cavity loss in our model, which is expected to increase because of the coupling of the central system to external metallic leads; therefore, we restrict our calculation to the transient time regime of less than 300 ps.

There are two main reasons for selecting a parallel double quantum dot system here. First, they can capture states with two-electrons that have a rich spectrum of collective oscillations


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that can be excited by the transport. Second, in order to observe Rabi and collective oscillations, we need the system to be weakly contacted to the leads for the initial state to be slowly decaying. For this purpose, the two-electron states in parallel dots are particularly convenient because their coupling to the leads is highly tunable.[24]

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### Notes

The authors declare no competing financial interest.

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